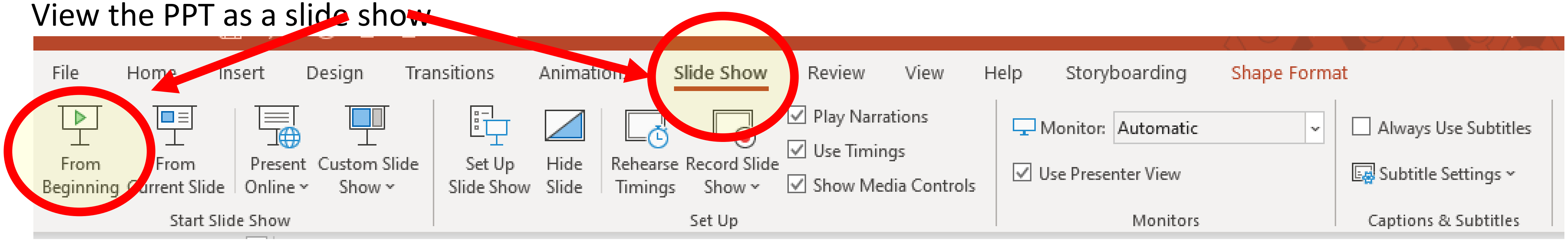
# How to best use these slides…

- View the PPT as a slide show




- Then click through every step
  - Mouse clicks will advance the slide show
  - Left/right arrow keys move forward/backward
  - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, use PS discussion boards, email me, and/or visit us in a Teams class session!

# LESSON 7.2b

## Graphing Translated Rational Functions

**Today you will:**

- Learn how translations apply to rational functions
- Graph translated rational functions
- Practice using English to describe math processes and equations

**Core Vocabulary:**

- Rational function, p. 366

**Previous**:
- Domain
- Range
- Asymptote
- Transformations
    - Horizontal and vertical shifts (translation)
    - Horizontal and vertical scales (stretches and shrinks)
    - Reflections over an axis

## Review of Transformations of Functions

| Transformation | f(x) Notation | Examples | |
|---|---|---|---|
| **Horizontal Translation**<br>Graph shifts left or right. | $f(x-h)$ | $g(x)=\sqrt{x-2}$<br>$g(x)=\sqrt{x+3}$ | 2 units right<br>3 units left |
| **Vertical Translation**<br>Graph shifts up or down. | $f(x)+k$ | $g(x)=\sqrt{x}+7$<br>$g(x)=\sqrt{x}-1$ | 7 units up<br>1 unit down |
| **Reflection**<br>Graph flips over $x$- or $y$-axis. | $f(-x)$<br>$-f(x)$ | $g(x)=\sqrt{-x}$<br>$g(x)=-\sqrt{x}$ | in the $y$-axis<br>in the $x$-axis |
| **Horizontal Stretch or Shrink**<br>Graph stretches away from or shrinks toward $y$-axis. | $f(ax)$ | $g(x)=\sqrt{3x}$<br>$g(x)=\sqrt{\frac{1}{2}x}$ | shrink by a factor of $\frac{1}{3}$<br>stretch by a factor of 2 |
| **Vertical Stretch or Shrink**<br>Graph stretches away from or shrinks toward $x$-axis. | $a \cdot f(x)$ | $g(x)=4\sqrt{x}$<br>$g(x)=\frac{1}{5}\sqrt{x}$ | stretch by a factor of 4<br>shrink by a factor of $\frac{1}{5}$ |

Also called Horizontal Scale and Vertical Scale

*Take a few minutes to review this*

**Let's go back and look at inverse variation functions…**

Consider our example from yesterday: $f(x) = \frac{4}{x}$

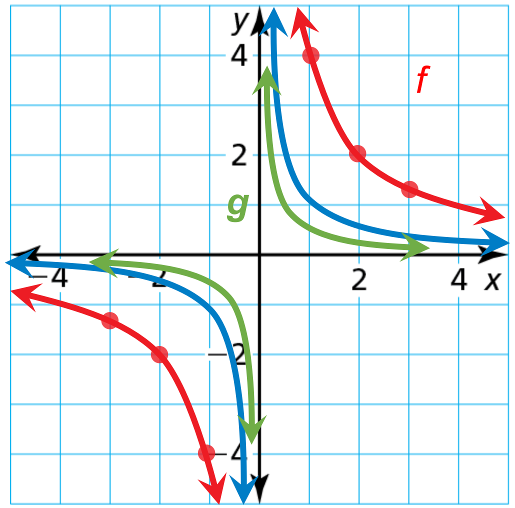
*Have any transformations been applied to it (when compared to its parent function)?*

- Hint: rewrite $f(x)$ showing the exponent on $x$
- $f(x) = 4x^{?} = 4x^{-1}$
- What transformation is the 4 (the constant of variation) applying? It is scaling $f(x)$
- $f(x)$ is being scaled vertically (a vertical stretch) by a factor of 4.

**Clarifying note:**
Rewriting the inverse variation function showing the exponent may lead some of us to think it is now a direct variation function. That is NOT true!
**Remember**: a negative exponent really means the variable flips to the "other side" of the fraction so here the $x$ is still in the denominator.

***Let's compare the graph of $f(x)$ to its parent function to see what the scale looks like***

- Here you can see the parent in blue and $f(x)$ in red
- You can see that $f(x)$ is pulled out and away from the parent
- It is "stretched" away from the "center" of the hyperbola (in this case the origin)
- What do you think $g(x) = \frac{1}{2x}$ would look like, where the scaling factor (shrink) is $\frac{1}{2}$?
- You can see it in green … it pulls in (shrinks) toward the "center" of the hyperbola



## Translations of simple rational functions

We've seen what scaling looks like for simple rational functions…

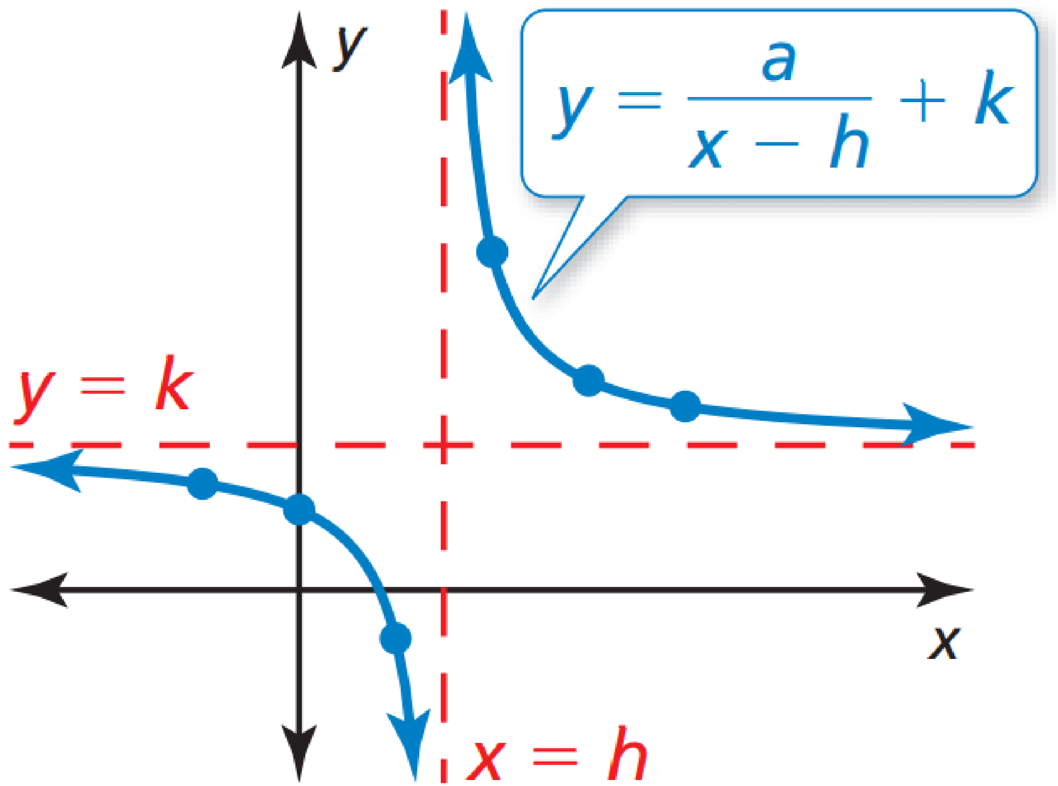
…let's see what ***translations*** (horizontal and vertical shifts) look like.

***One quick but important note…***

- When we translate the function, what do you think happens to the asymptotes?
- They translate/shift with the function!
- Now it will be necessary to identify and plot the asymptotes

## Translated rational functions

- Generic form: $y = \frac{a}{x-h} + k$
- $h$ is the horizontal shift (subtracting moves right, adding moves left)
- $k$ is the vertical shift (adding moves up, subtracting moves down)
- Horizontal asymptote: $y = k$
- Vertical asymptote: $x = h$
- Shifts the "center" of the hyperbola



**Update – how to graph rational functions**

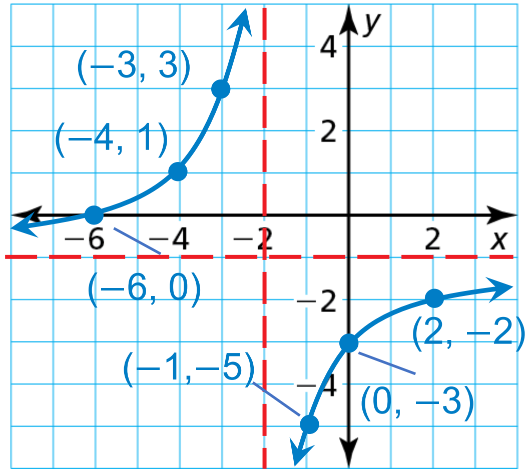
1. Draw the asymptotes
   - If translated: vertical asymptote is $x = h$, and horizontal asymptote is $y = k$
   - If not translated: the $x$ & $y$ axes are the asymptotes
2. Plot points to the left and to the right of the vertical asymptote
   - Pick numbers that are easy to calculate and to plot
   - If $a$ is negative, the graph will be reflected around the $x$ axis
3. Connect the dots
   - Draw the branches so they approach but do not touch the asymptotes

Graph $g(x) = \dfrac{-4}{x+2} - 1$. State the domain and range.

SOLUTION

**Step 1** Draw the asymptotes $x = -2$ and $y = -1$.

**Step 2** Plot points to the left of the vertical asymptote, such as $(-3, 3)$, $(-4, 1)$, and $(-6, 0)$. Plot points to the right of the vertical asymptote, such as $(-1, -5)$, $(0, -3)$, and $(2, -2)$.




**Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

## LOOKING FOR STRUCTURE

Let $f(x) = \dfrac{-4}{x}$. Notice that $g$ is of the form $g(x) = f(x - h) + k$, where $h = -2$ and $k = -1$. So, the graph of $g$ is a translation 2 units left and 1 unit down of the graph of $f$. Also note the reflection over the $x$-axis since $a = -4$

The domain is all real numbers except −2 and the range is all real numbers except −1.

## Review/Recap

Translations keep coming back…

- Make sure you are reviewing these periodically
- Once you understand how translations work for a simple function family, the same applies to more complex function types!

Translations for Rational Functions

- Inverse variation functions are an example of scaling … $a$ (the constant of variation) is the scaling factor
- Generic form: $y = \frac{a}{x-h} + k$
- $h$ is the horizontal shift (subtracting moves right, adding moves left)
- $k$ is the vertical shift (adding moves up, subtracting moves down)
- Horizontal asymptote: $y = k$
- Vertical asymptote: $x = h$
- Shifts the “center” of the hyperbola

Khan Academy

- I’m not recommending the Khan Academy videos for this stuff, he covers things we are not…

# Homework

Pg 370, #11-24